Quiz 5.1: Sample Answers

1. Find $\frac{dy}{dx}$ for $50\sqrt{x} - 5\sqrt{y} = 25$.

   Taking the derivative of both sides, we get:

$$50\left(\frac{1}{2}x^{-0.5}\right) - 5\left(\frac{1}{2}y^{-0.5}\right)\left(\frac{dy}{dx}\right) = 0$$

   Then solving for $\frac{dy}{dx}$, we get:

$$\begin{aligned}
\left(\frac{-5}{2}y^{-0.5}\right)\left(\frac{dy}{dx}\right) &= \left(\frac{-50}{2}x^{-0.5}\right) \\
\frac{dy}{dx} &= \frac{\frac{-50}{2}x^{-0.5}}{\frac{-5}{2}y^{-0.5}} \\
\frac{dy}{dx} &= \frac{10x^{-0.5}}{y^{-0.5}} \\
\frac{dy}{dx} &= \frac{10y^{0.5}}{x^{0.5}} \\
\frac{dy}{dx} &= \frac{10\sqrt{y}}{\sqrt{x}}
\end{aligned}$$

2. Find $\frac{dy}{dx}$ for $2\sin x + 2\cos y = 4$.

   Taking the derivative of both sides, we get:

$$2\cos x - 2\sin y\left(\frac{dy}{dx}\right) = 0$$

   Then solving for $\frac{dy}{dx}$, we get:

$$\begin{aligned}
-2\sin y\left(\frac{dy}{dx}\right) &= -2\cos x \\
\frac{dy}{dx} &= \frac{-2\cos x}{-2\sin y} \\
\frac{dy}{dx} &= \frac{\cos x}{\sin y}
\end{aligned}$$

3. Find $\frac{dy}{dx}$ for $y = -\arctan(2x^2)$.

   Since the derivative of $\arctan x$ is $\frac{1}{1+x^2}$, we have:

$$\frac{dy}{dx} = (-1)\frac{1}{1+(2x^2)^2}(4x) = \frac{-4x}{1+4x^4}$$

4. Find $\frac{dy}{dx}$ for $y = -[\arctan(2x^2)]^2$.

   We apply the chain rule twice to get:

$$\frac{dy}{dx} = -2\arctan(2x^2)\left(\frac{1}{1+(2x^2)^2}\right)(4x)$$

   Simplifying gives:

$$\frac{dy}{dx} = -\arctan(2x^2)\left(\frac{8x}{1+4x^4}\right)$$

5. Find the equation of the tangent line to the curve $x^2 + 2y^2 = 36$ at the point $\left(\frac{1}{2}, \frac{\sqrt{286}}{4}\right)$.

   Taking the derivative and simplifying, we get:

$$\begin{aligned} 2x + 4y(y') &= 0 \\ y' &= \frac{-2x}{4y} \\ y' &= \frac{-x}{2y} \end{aligned}$$

   Substituting $(x, y) = \left(\frac{1}{2}, \frac{\sqrt{286}}{4}\right)$, we get

$$y' = \frac{\frac{-1}{2}}{2\frac{\sqrt{286}}{4}} = \frac{-1}{\sqrt{286}}$$

   We then substitute this slope m into the equation $y = mx + b$ to solve for b:

$$y = mx + b \Rightarrow \frac{\sqrt{286}}{4} = \frac{-1}{\sqrt{286}}\frac{1}{2} + b \Rightarrow b = \frac{72}{\sqrt{286}}$$

Thus the equation of the line is

$$y = \frac{-x}{\sqrt{286}} + \frac{72}{\sqrt{286}}$$